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ABSTRACT

This report discusses the change in mathematics curriculum influenced by researchers, society, and the mathematics field. Many research results suggest that students should learn mathematics by experience. Students' misconceptions may be formed because of the sudden shift from arithmetic to algebra. Up to the eighth grade, mathematical content has been heavily repeated. For more of the active doing of mathematics, four common activities (abstracting, inventing, proving, and applying) and four processes (generalizing, conjecturing, convincing, and specializing) are recommended. The curriculum standards developed by the National Council of Teachers of Mathematics, the curriculum framework developed by the National Academy of Sciences, some projects sponsored by the National Science Foundation, the University of Chicago School Mathematics Project, and other projects are described. Constraints against curricular change are listed and teacher collaboration is emphasized as accessary to obtain change in mathematics curriculum. Six schools selected as exemplary programs are introduced. Addresses of five mathematics curriculum projects and project directors involved in the Urban Mathematics Collaborative project are listed. (YP)

ASCD CURRICULUM UPDATE

Transforming the "Underachieving" Math Curriculum

Mark Driscoll

Developing Leadership for Quality in Education

ASCD

Curriculum Update

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Transforming the 'Underachieving' Math Curriculum

By Mark Driscoll

In 1986, the state of California appealed to its own recently delineated mathematics framework and, in one decisive stroke, rejected all of the elementary-school math textbooks submitted for state adoption. Shortly thereafter, in early 1987, the disappointing American test results from the Second International Mathematics Study (SIMS) were announced, and American SIMS researchers pointed an accusing finger at our "underachieving curriculum."[1]

To those close to the soil in math education, these were more than just two more stories of malaise on the American education scene; they were clear signs of sweeping changes to come. At the same time, they were not especially surprising, since calls for broad change have been building for years, in several very different quarters.

The strongest impetus for change comes from research. Researchers in both

About the Author

Mark Driscoll is a project director at Education Development Center (EDC) in Newton, Mass. In addition to experience in teaching and curriculum development, Driscoll directed A Study of Exemplary Mathematics Programs at NEREX, Inc., in Chelmsford, Mass., from 1982–84. At EDC, he directs the technical assistance to the Urban Mathematics Collaborative Project and assists in managing the Reckoning With Mathematics Project, both described in this text. He can be reached at EDC, 55 Chapel St., Newton, Mass., (617) 969-7100.

Experts say students need to become more involved in "doing," rather than "learning about" mathematics Students at Weston High School outside Boston using "Geometric Supposer" software developed by the Education Development Center.

Photos by Education Development Center

As part of a pilot test of the Education Development Center's Reckoning With Mathematics project, fourth graders in Cambridge, Mass., work with three-dimensional shapes to begin to conceptualize the notion of volume.

Researchers in both education and cognitive psychology have uncovered numerous flaws in the way math is usually presented to students in the classroom: flaws in scope, in sequence of topics, and most importantly, in the psychological assumptions about how math is learned.

education and cognitive psychology have uncovered numerous flaws in the way math usually is presented to students in the classroom. flaws in scope, in sequence of topics, and, most importantly, in the psychological assumptions about how math is learned.

Second, society itself has changed. We are citizens of the Information Age, constantly barraged with data that require gathering, organizing, representation, and predictive manipulation: all skills that get scant attention in the present math curriculum.

And third, the math field has been transformed in the past ten years—a transformation swept along, in good part, by the computer. In almost dizzying fashion, mathematicians have been tapping the power of computers to help them probe new ideas and approximate solutions to both old and new problems.[2] Yet, while the calculator and computer have made it possible to bring elementary and secondary school students into closer range of these powerful ideas, their contribution in this direction has been minimal.

The case for change in the school math curriculum is compelling, yet the obstacles to change are considerable. For that reason, the landscape in math curriculum development is an especially dynamic one right now. This report will attempt to capture, with a few wide snapshots, the look of that landscape.

The Case From Research

The most compelling case for change in the math curriculum is from research. Further, if there is one phrase that drives the research argument, it is that students learn math by **doing** math.

Thomas Romberg, a researcher in math education at the University of Wisconsin, points back to the distinction made by John Dewey between "knowledge" and "the record of knowledge." According to Romberg:

> The distinction between knowledge and the record of knowledge is crucial. A person

gathers, discovers, or creates knowledge in the course of some activity having a purpose: this active process is not the same as the absorption of the record of knowledge—the fruits of past activities. When the record of knowledge is mistakenly taken to be knowledge, the acquisition of information becomes an end in itself, and the student spends his time absorbing what other people have done, rather than having experiences of his own.[3]

No longer can we assume that students learn math well by listening to the information passed on to them by their teachers and then putting into practice what they have heard in rote fashion. There is too much evidence of deep and persistent misconceptions developed by students at all levels, even many students who score well on standardized tests: misconceptions about operations like multiplication of whole numbers, about fractions, about algebraic concepts like "variable."

Unlike routine errors that reveal themselves rather transparently on written examinations, these misconceptions can be both subtle and insidious.

Magdalene Lampert, a researcher in education at Michigan State University, also teaches math in an elementary school in the East Lansing area. Her research has shown that, while young students are able to attach some meaning to simple, single-digit multiplication—i.e., they recognize it as a means of recording repeated additions of a number—many of them lose that sense of meaning when they move on to multiplication of multidigit numbers. They may learn a computational algorithm well, and even use it correctly, but all meaning intended by text and teacher has been drained from the computation.

In place of the intended meaning, students construct a meaning that has little to do with the usefulness of multiplication. For example, they may picture math as a collection of procedures, and conclude that multiplication is merely one of the procedures that has to be learned. In the process, many students never sharpen their judgment about real-world situations in which multiplication is appropriate. In this, as in so many other cases in the curriculum, little is done to help students to construct a more powerful and useful meaning for the concept.[4]

Transition to Algebra

For older students entering algebra for the first time, misconceptions often sneak into their understanding about the meaning of equations. Insofar as any rationale for learning algebra is offered students, it is that algebra is a kind of generalized arithmetic, a symbolic language that will permit them to make general statements and investigations about numbers. Yet, the understanding of equation necessary to do algebra is different from the understanding of equation necessary to do arithmetic.

In arithmetic, an equation is most often an action statement—e.g., do this action to these numbers and you get that answer. But, in algebra, the notion of equivalence is quite often in central focus—e.g., you can make the same change to each side of the equals sign, and the equivalence does not change.

Researchers believe that the sudden shift in focus is confusing for many students and interferes with their learning. After all, they have spent years constructing in their minds a meaning for the notion of equation, and suddenly comes a new set of challenges for which the constructed meaning is next to useless. Clearly, in the chain of the math curriculum, the shift from arithmetic to algebra is one of the weakest links.

A conclusion from this research is that in learning math, students construct understanding individually. Often, that constructed understanding is skewed in one direction or another from what teachers and curriculum developers intend. As a result, the curriculum must be rebuilt so that the doing, or constructing, of math comes to the surface and becomes an open part of classroom instruction. Students must experience the "knowing how" as well as the "knowing that" in math.

Considerable Repetition

Will this new focus swell an already tightly packed curriculum? Not necessarily. There is some question as to how tightly packed the current math curriculum really is. For example, the data from the Second International Mathematics Study provide persuasive evidence that the the curriculum cyclically repeats information about difficult concepts and skills. A student who is nudged in the wrong direction on the first pass through a difficult math concept or skill receives little benefit from a second or third pass if they resemble the first.[5]

Yet the present curriculum is filled with such futile revisiting of math concepts and skills. The original intent of the "spiral curriculum" developed by Jerome Bruner—that each "revisiting" should be deeper and more complex—has been lost. Consequently, in the words of the SIMS researchers: "The U.S. mathematics curriculum is characterized by a great deal of repetition and review, with the result that topics are covered with little intensity."[6]

In a detailed analysis of three of the most popular elementary school textbooks, James Flanders of the University of Chicago gave dramatic proof of the pervasiveness of repetition in the math curriculum.[7] Flanders put pages from the texts into two categories, new and old, and defined a page as "new" if it contained even *one* idea or type of exercise that was not covered the previous year in the same text series. He then counted the pages in each category.

University of Chicago Math Project

The University of Chicago School Mathematics Project (UCSMP), with primary funding from Amoco and further support from several other foundations, is producing a complete curriculum for grades 7–12, with a target population of the middle 80 percent of students. Among its striking variations from the status quo:

- The learning of algebra is moved earlier—for most students, in grades seven or eight;
- Geometry is woven throughout;
- The calculator is used regularly in all grades;
- The computer is an integral part of students' development of statistical thinking and an understanding of functions.

Despite such a generous definition of newness, the texts showed a remarkable amount of repetition. For example, the average percent of new content in the three text series was less than 40 percent in grades 6 and 7 and only 30 percent in grade 8. Earlier grades were similar.

The major exception to the pattern of repetition is the transition from eighth grade to ninth grade, when almost 90 percent is new to students in ninth-grade algebra texts. This must be one of the rudest awakenings in all their school years, yet the cause is, quite simply, faulty curriculum design.

As the SIMS researchers point out: The eighth grade mathematics curriculum in the U.S. tends to be 'arithmetic-driven, resembling much more the end of elementary school than the beginning of high school.'[8] The sudden shift outward from a narrow arithmetical view of math leaves students prone to serious misconceptions.

[illegible] National Projects [illegible] Under Way

Two national efforts currently are under way to transform the way math is taught in the nation's schools.

The Curriculum Standards Committee appointed by the National Council of Teachers of Mathematics released its preliminary [illegible] recommendations last October, now circulating in the math education community for comment. The new goals for math in grades K-12 provide strong support for the [illegible] that students must [illegible]

- become [illegible]
- learn to communicate mathematically
- learn to reason [illegible]
- value mathematics [illegible]
- develop confidence [illegible] ability to do math [illegible]

Meanwhile, the [illegible] Sciences Education Board [illegible] of the National Academy of [illegible] is attempting to [illegible] math curriculum [illegible] The effort [illegible] education [illegible] but [illegible] mathematics [illegible]

[illegible] to [illegible] entrenched opposition [illegible] NCTM's draft statement notes that "the administrator who has charted group scores on the standardized test that has been given in the district for years is going to be reluctant to replace it," according to a report in *Education Week*.

The Need for Change

Because the math curriculum is woefully repetitive, at least up to the ninth grade, it could make room for more of the active **doing** of math. What can be done to infuse this into the curriculum? A reasonable first step is to make sure that curriculum developers understand what it means to do math. There appears to be a concise way to describe that process, as illustrated in the two very similar analyses offered by Romberg and Leone Burton, a researcher in mathematics education at London's Avery Hill College.[9]

Romberg writes that "it is easy to recognize four related activities common to all of mathematics: **abstracting, inventing, proving, and applying.**"[10] To cut into this list a bit more deeply, there are two types of abstractions: concepts and procedures. An example of a conceptual abstraction already mentioned is the notion of equation. And the algorithm for multiplying multidigit numbers, also mentioned earlier, is an example of a procedural abstraction.

Burton distinguishes four processes involved in doing math: **generalizing, conjecturing, convincing, and specializing.**[11] Generalizing implies movement from the specific to the general. The concept of variable in algebra results from such a process. Conversely, specializing implies movement from the general to the specific—e.g., recognizing or creating examples of general math concepts.

Clearly, Burton's list compares comfortably with Romberg's list, and nearly maps onto it in one-to-one fashion. The point in mentioning both lists is that, once committed to infusing the math curriculum with more "doing," curriculum developers can be assured that there are few categories to attend to.

As an example of how this infusion might work, consider again the multidigit multiplication issue raised before. Lampert has dealt successfully with the empty meaning attached to the multiplication algorithm by her fourth graders in the following way. She presents them with a multidigit exercise—e.g., 78 x 45—and asks them to invent a story, or stories, for which this multiplication provides the answer.[12]

She then has them illustrate the story, and thus the multiplication, pictorially (using groupings of stick figures, for example). In the process, students convince the teacher and each other of the correctness of their pictures.

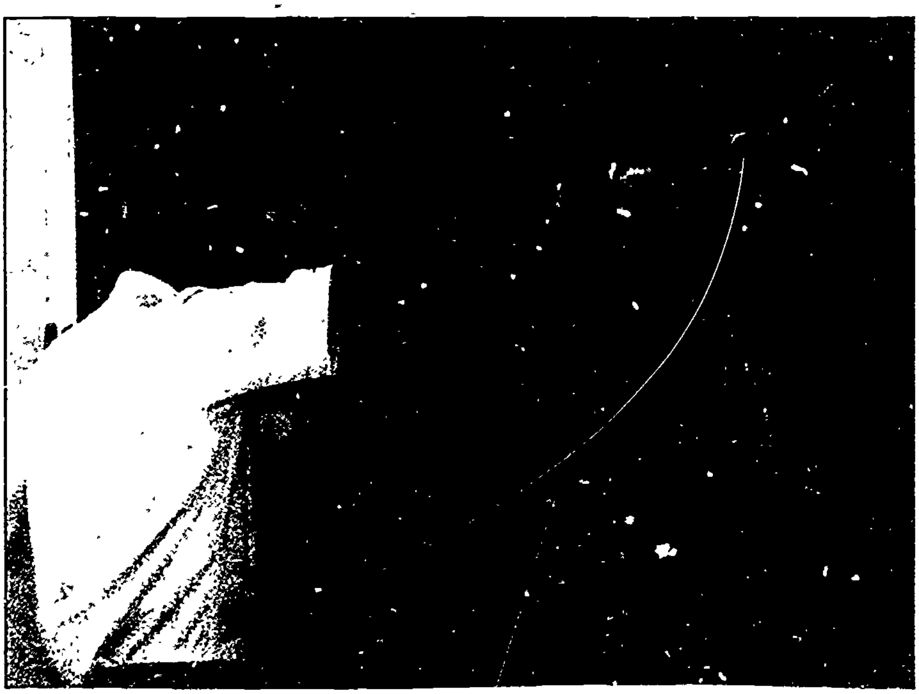
Since defending the match between story and multiplication is essential, each student is constructing a meaning for the algorithm that is connected to the application of math in the real world. In this simple but elegant classroom experiment, there are at least invention, application, and convincing in what students are doing. Clearly, this alternative, student-centered approach marks a direction in which curriculum developers should look as they chart the new mathematics curriculum.

What Course to Chart?

Of course, math education is conducted in a world where politics and conformity play crucial roles in decisions that are made. No matter how persuasive research results may be, real change in math education will not happen unless there are parallel efforts to influence policy and create comprehensive alternatives to the existing curriculum.

Two ambitious and complementary efforts to affect policy have been launched by the National Council of Teachers of Mathematics (NCTM) and the National Academy of Sciences (NAS).

NCTM's Curriculum Standards Committee has spent the past year revising the set of standards used to guide the



construction of school math programs. Their preliminary recommendations appeared in October 1987 and are currently circulating in the math education community for comment.

Five general goals for students are woven throughout NCTM's math curriculum standards for grades K-12, and they herald a new attitude about what is important in math education:

Becoming a mathematical problem solver

Learning to communicate mathematically

Learning to reason mathematically

Valuing mathematics

Developing confidence in one's ability to do math

The NAS, which sponsors the Mathematical Sciences Education Board (MSEB), has embarked on an even larger task. It includes not only math educators from the precollege level but university and industrial mathematicians as well. While the NCTM effort is aimed at a curriculum for 1990, MSEB will explore the math curriculum for the year 2000.

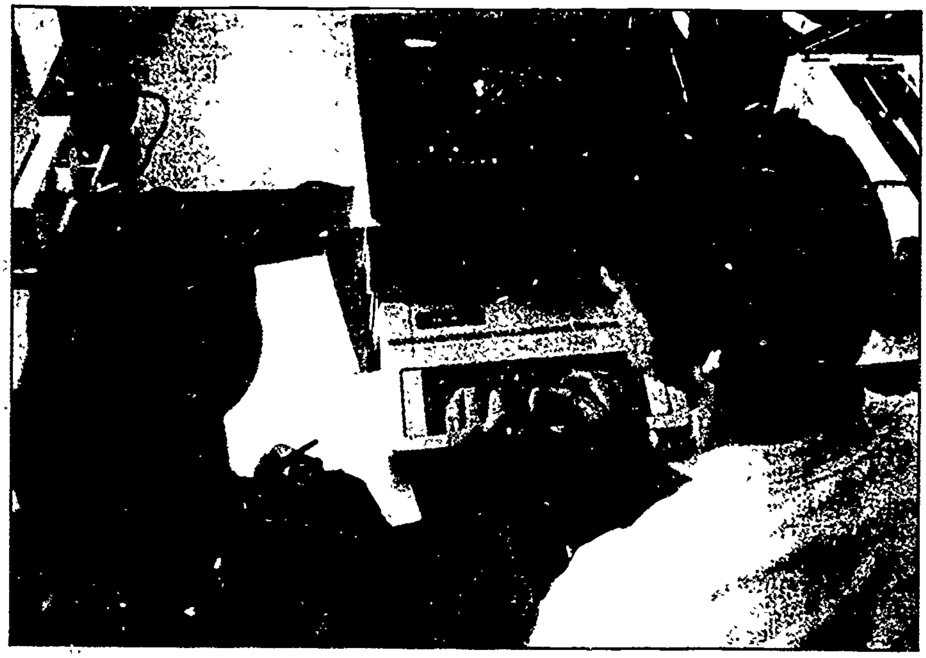
It is expected that the two efforts will dovetail nicely in providing policymakers with a clear alternative to the present outdated math curriculum, one that is in line with research recommendations for more student-centered teaching and learning, with a healthy infusion of problem solving and real (as opposed to contrived) applications

Projects aimed at improving math education in the early grades are seen as pivotal in the coming decade. Anthony Ralston, professor of mathematics and computer science at the State University of New York/Buffalo and chair of MSEB's Curriculum Framework Committee says, "The more I am involved in this work, the more I am convinced that elementary school problems transcend other problems.

"Unless we make considerable changes at that level," Ralston continues, "changes in the later grades will have little meaning. In particular, we need all the leadership we can muster to end the dominance of paper-and-pencil computation in the early grades, and to integrate the use of calculators and computers."

Development Projects

There also is reason to expect that the alternative course recommended will be in line with some large development projects currently under way. The following set of examples is not intended to be a comprehensive accounting of innovative projects; in fact, the proliferation of novel development projects is cause for considerable optimism in math education.



The National Science Foundation (NSF) is supporting a family of six four-year projects to find meaningful and convincing ways to integrate the calculator and computer into the elementary school curriculum. NSF also supports the Middle Grades Mathematics Project, developed at Michigan State University, which seeks to enrich the curriculum for a set of grade levels where the curriculum traditionally has included considerable repetition.

Finally, there is *Square One Television*, Children's Television Workshop's innovative math series for upper elementary school and middle school students, which can and should be used in close conjunction with the revamped school curriculum. It also serves as a way to engage parents in the dialogue about curriculum change.

The Education Development Center, based in Newton, Mass., is developing one of the family of six NSF projects, entitled Reckoning With Mathematics. A brief description of some of its goals might lend substance to visions of the future elementary school curriculum.

Primarily, the project intends to show how the curriculum can be restructured, in scope, sequence, and pedagogical approach, into a series of interlocking yet independent modules. Typically, a module spans a two- to four-week period, during which students are immersed in doing math in all of the ways described earlier. inventing, convincing, abstracting, etc.

The structure of each module comprises a setting or settings rich in mathematical potential, a collection of challenges that allow teachers and students to plumb that potential, and a set of tools with which students can take on the challenges. These tools include both the traditional, like manipulatives and rulers, and the relatively new, such as the calculator and computer.

In one prototype module, for example, students are immersed in a geometry setting defined by scale drawing. The tools that are brought to bear on the challenges include rulers, graph paper, geoboards, and a newly developed electronic geoboard that can be used on the computers most commonly found in schools today.

Focus on Statistics

The other NSF projects in the family are exploring different, though complementary, approaches to revamping the curriculum.

"The U.S. mathematics curriculum is characterized by a great deal of repetition and review, with the result that topics are covered with little intensity."
—Second International Mathematics Study

For example, Used Numbers, a project being developed by the Technical Education Research Centers and Lesley College in Cambridge, Mass., will make statistical thinking accessible to elementary school students.

At the secondary level, there are several projects showing the way toward a new curriculum. As at the lower-school level, the major projects do not diverge much in their visions of math education.

For example, the University of Chicago School Mathematics Project (UCSMP), with primary funding from Amoco and further support from several other foundations, is producing a complete curriculum for grades 7–12, with a target population of the middle 80 percent of students. Among its striking variations with the status quo, the learning of algebra is moved earlier (for most students, in grades seven or eight), geometry is woven throughout, the calculator is used regularly in all grades, and the computer is an integral part of students' development of statistical thinking and an understanding of functions.

While UCSMP has been developing its curriculum, the North Carolina School of Science and Mathematics, with funding from the Carnegie Corporation, has begun an innovative and computer rich fourth year course for high schools. It is comprised of six modular units, each of which provides students with a setting filled with up-to-date math.

Finally, the College Board's Project EQuality has brought together representatives of both high school and college math to produce a set of recommendations for the kind of curriculum that will best serve students who are preparing for college math.[14]

The Bridge to Change

With history as guide, one can safely bet that none of the above projects will become widely adopted without resistance. The basic conservatism of textbook publishers will slow down change, but so will several other factors.

For one thing, schools as well as publishers will require firm and concise statements of math learning objectives for each grade level, and test makers will require time and guidance to learn how to evaluate these new and challenging learning objectives. It is one thing to evaluate whether students can solve a certain type of algebraic equation, it is quite another to evaluate how well students are inventing, generalizing, abstracting, or otherwise **doing** math.

Similarly, if the "underachieving curriculum" is to move significantly away from the current model, then the paradigm of teaching math also must change significantly. Class periods dominated by lecture and silent student practice can no longer be the rule.

New models of teacher training also are needed to match the changed paradigm of teaching. Whatever their design, the new models must make it possible for teachers to step back and transform their basic beliefs about what goes into the teaching and learning of math. They must be able to do for teachers, on a large scale, what Eleanor Duckworth and colleagues at the Massachusetts Institute of Technology were able to do for twenty elementary-school

Exemplary Programs Spur Student Interest

The following schools were selected from a number of case studies profiled in *Stories of Excellence: The Study of Exemplary Mathematics Programs*, from which the profiles were drawn, was funded by the U.S. Department of Education and continued from 1982–84.

Lawton Elementary School, Ann Arbor, Mich.

Lawton's math program is distinguished by a liveliness that extends beyond the classroom walls. Students participate in activities such as a math club and newsletter; teachers share materials and ideas; and the principal clearly communicates the commitment to a central role for non-traditional math problem solving. The school uses the Comprehensive School Mathematics Program, and teachers adapt and supplement it when they see fit. The result is a curriculum in which all students receive at least 55 minutes of math each day. Staff collegiality, at least in the sense of sharing, is exceptionally strong. Leadership from the district is supportive, and complements the principal's leadership at the school level.

Franklin Middle School, Nutley, N.J.

Franklin Middle School (grades 7–8) is distinguished by a math program in which nationally standardized test scores have vaulted from mediocre to excellent in the past decade. The superintendent has been the catalyst and overall architect; the principal infuses the school with a serious, yet supportive, tone and high expectations; the guidance staff carefully monitors student placement and parent communication; and teachers are active participants in the redesign of the curriculum while, in the classroom, they maintain an effective blend of support and high expectations.

Huron High School, Ann Arbor, Mich.

A spirit of staff teamwork at Huron High School has led to a multifaceted approach to excellence in the math program. Students in the lowest-level courses receive careful attention and innovative course materials; minority students benefit from a vibrant supplementary pre-engineering program; and the staff makes a conscious effort to keep girls from dropping math courses. The overall percentage of students taking four years of math is exceptionally high. Computers and teacher-developed software are used extensively.

Rufus King High School, Milwaukee, Wis.

Rufus King High School is an urban magnet school for college-bound students. Its math program has incorporated the International Baccalaureate Program for the top courses, and is distinguished by a cohesive, sharing staff of teachers who commit considerable time. Leadership is strong throughout—the department head acts consistently in support of teachers, and the principal and higher administrators give the program the underpinnings it needs. With more than 50 feeder schools, the program faces considerable challenges to respond to student needs, and from the staff's desire to keep expectations high.

Stuyvesant High School, New York, N.Y.

Stuyvesant is a specialized public high school for talented students. Student interest in math extends far outside the classroom, with daily meetings of math clubs and numerous extracurricular research projects. Teachers consistently communicate their expectations of clarity in students' mathematical arguments. Teachers also use coaching techniques in classes. The department head supports teachers in obtaining necessary resources, often in the face of a daunting lack of bureaucratic response.

North Carolina School of Science and Mathematics, Durham, N.C.

The North Carolina School of Science and Mathematics is a statewide boarding school for students in grades 11 and 12 who are talented in these fields. The math curriculum ranges from second-year algebra to second-year calculus, and students are carefully placed in appropriate courses. Teachers regularly observe and consult each other. Department members carefully nurture a "teacher-as-learner" environment and work to develop innovative curriculums.

teachers in their project, An Experiment in Teacher Development.[14]

In that project, variations on traditional themes like base-ten place value and the long-division algorithm led to a set of interesting and relatively deep math investigations on the part of the teachers, which compelled them to reflect carefully on how they learn and, more importantly, on how their students learn math. It is no mean challenge, but new teacher-training models will have to engage teachers in similar reflective experiences.

Individual Efforts

In the past five years, I have been involved in two projects that have revealed examples of how individual teachers and schools and groups of teachers in the same district can take steps to improve the math curriculum.

The first project, A Study of Exemplary Mathematics Programs (1982–84), was designed to study factors and conditions associated with excellence in student outcomes in precollegiate math. One of the powerful factors that emerged was **staff collegiality**. Teachers in the schools worked together to define a vision and expectations far beyond what is normal in American math education, namely, isolated teachers left to implement the curriculum alone and, most often, not looking beyond their textbooks.[15]

At an elementary school we visited in Ann Arbor, Mich., for example, teachers conducted a problem-solving session once a week for fifth-graders that was supplementary to their classwork. They invited occasional speakers to talk with the children about how math plays a part in such familiar phenomena as electricity. They also produced a summer math newsletter mailed home to all students. In short, they created an environment in which students experienced math as something that touched their lives well beyond the fifty or so minutes of daily classroom instruction.

Similarly, as described in the set of case studies from the study, we saw several high schools where environments were nurtured to support and encourage the involvement and interest of students in math outside the classroom, and where classroom lessons consistently valued student questioning and also put a high premium on clarity in student explanation and proof.[16]

Teachers Collaborate

If that study revealed that math curriculum and teacher culture can be richly intertwined, a second, newer project has convinced me that they ought to be, that is, to an extent without much precedent in American education, curriculum change should flow out of a collegial teacher culture where teachers plan and set visions together

In the Urban Mathematics Collaboratives Project, supported by the Ford Foundation, math teachers in eleven cities have begun some innovative projects with colleagues from business, academia, and other mathematics-using institutions in their cities.[17] The projects vary in detail and texture from city to city, but all have brought teachers together in a collaborative attempt to reconceive math teaching and to develop new and viable models of the profession.

Some of the collaborative activities are directed at enriching the curriculum For example, in Cleveland and Los Angeles, teachers and engineers from local firms have developed sets of engineering problems that may be used in high school math. In San Francisco, teachers and staff members at the Exploratorium science museum have collaborated on materials that highlight the math partially hidden in some of the physics exhibits in the museum. And in all eleven cities, teachers have made use of the collaborative to explore the uses of new computer software that might otherwise be slow to reach the majority of classrooms in their large, urban districts.

Conclusion

Curriculum, as John Goodlad has described it, is not a unitary concept: its meaning changes according to context and perspective. Among its several forms, there are the ideal curriculum, the implemented curriculum, the taught curriculum, the learned curriculum, and the tested curriculum. Unfortunately, in most school districts in the United States, the differing contexts and perspectives usually lead to a harmful lack of coordination among these embodiments.

As this report attempts to show, substantial changes in learning theory, in math, and in technology, to name three of the most influential factors, have made it imperative that sweeping changes occur in the math curriculum at all school levels, and a parallel effort must be made to coordinate the elements of the curriculum that are implemented, taught, learned, and tested.

The challenge is daunting, yet we have no choice but to try if we want math education in this country to regain its pre-eminence in the world. The prospects —in the form of innovative curriculum development projects and of innovative attempts to restructure math teaching so that teachers will lead the reform effort—appear very, very good.

No matter how persuasive research results may be, real change in math education will not happen unless there are parallel efforts to influence policy and create comprehensive alternatives to the existing curriculum.

Endnotes

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Correction

For information concerning resources published by Network Publications referred to on pp. 11–12 in the October 1987 issue of *Curriculum Update*, contact:

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